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Sources of Time Variation in the Covariance Matrix of Interest Rates*

The main objective of this paper is to study the sources of time variation in the covariance matrix of interest rates. We depart from the traditional standard deviation–correlation decomposition of covariances and investigate whether time variation in the covariance matrix of bond yield changes is caused by time-varying eigenvalues and/or eigenvectors. On the basis of a formal testing procedure, we find that common factors display a clear time-varying volatility over the past three decades. Most notably, we observe that the switches in monetary policy that take place with the appointment of a new Federal Reserve chairman play an important role in characterizing the time variation in the loadings on the common factors that drive interest rates.

I. Introduction

To the extent that economic and political conditions do change over time, one would expect the volatility of interest rates to change as well. For instance, numerous studies identify significant fluctuations in the variability of U.S. short-term interest rates (see, e.g., Hamilton 1988; Ang and Bekaert 2002; Smith 2002) or in the variability of U.S. long-term interest rates (see Watson 1999). Over different time periods, the term structure of the volatility of bond yield innovations, the so-called volatility curve, looks different. Indeed, while the volatility curve has tended to be hump-shaped during the Greenspan era, the hump

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disappeared during the 1979–82 monetary experiment (see Piazzesi 2003) and was less distinct prior to the monetary experiment (see Dai and Singleton 2003). Moreover, besides volatilities, one would expect that comovements among interest rates would also change. For instance, Christiansen (2000) finds that correlation tends to increase on macroeconomic announcement days, and Wu (2001) documents how changes in monetary policy simultaneously affect volatilities and comovements between long and short rates.

In the present paper, we depart from the traditional standard deviation–correlation decomposition of covariances in order to study the sources of time variation in the covariance matrix of interest rates. Our approach is built around the fact that the covariance matrix of bond yields is often used in the finance literature as an input for principal component analysis in which the eigenvectors and the eigenvalues are jointly estimated (see Litterman and Scheinkman 1991). The main contribution of this paper is to present and perform statistical tests investigating whether time variation in the covariance matrix of bond yield changes is caused by time-varying eigenvalues and/or eigenvectors. Our approach is based on the common principal component (CPC) analysis, which extends the standard principal component analysis in the case of several populations (see Flury 1984, 1988).

Traditional principal component analysis provides much of the intuition for the dynamics of bond yields (see Piazzesi 2003). Empirical analysis generally determines that three principal components are needed to almost fully explain the dynamics of the term structure of interest rates.[1] The interpretation of these principal components in terms of level, slope, and curvature describes how the yield curve shifts or changes shape in response to a shock on a principal component.[2] These labels have turned out to be extremely useful in thinking about the driving forces of the yield curve until today and have important macroeconomic and monetary policy underpinnings (see Rudebusch and Wu 2003). Moreover, the latent factors implied by estimated affine term structure models typically behave like the first principal components (see Bams and Schotman 2003; Dai and Singleton 2003). For these reasons, the present paper provides some key insight into the stability of the latent factor structure of interest rates through time.

Some authors have already studied the time robustness of the factor structure of interest rates. For instance, Bliss (1997) breaks down his 1970–95 sample period into three subperiods and notices that factor loadings exhibit a consistent pattern across subperiods (see also Chapman and Pearson 2001). However, Phoa (2000) reports evidence that results on the curvature may be less robust than those on the level and slope. Moreover, there is evidence of increased volatility of the factors during particular periods, such as restrictive

1. For the postwar period, the first three principal components already capture over 96% of the total variation in U.S. bond yield changes; in the case of bond yield levels, the proportion is even higher, i.e., over 99.5% (see Piazzesi 2003, table 1).

2. Similar findings have been reported with factor analysis (see Bliss 1997).

monetary policy ones (see Bliss 1997). While the above studies are mainly exploratory in nature, our approach is based on a formal testing procedure.

In this paper, we pay particular attention to the links between the behavior of the covariance matrix of interest rates and changes in monetary policies. Two recent contributions motivate our approach. First, Piazzesi's (2005) analysis lends support to a monetary interpretation of the volatility hump. More precisely, she claims that the different shape of the term structure of bond yield volatilities can be explained in part by the varying degree of policy inertia under different Federal Reserve chairmen.[3] However, because her analysis is based on a subsample of the Greenspan term, one should be careful about treating this effect as a robust fact. Second, Dai, Singleton, and Yang (2003) show that the volatility curves are very different across high- and low-volatility regimes and that the well-known hump is mainly a low-volatility regime phenomenon. For these reasons, in the empirical section of the paper, we associate different subperiods with the terms of Federal Reserve chairmen and also distinguish between periods of low volatility, such as the Greenspan term, and periods of very high volatility, such as the first part of the Volcker term.

We apply our methodology to the U.S. term structure of interest rates over the past three decades. The data used in the empirical analysis are the Treasury zero-coupon bond yields from January 1970 to December 2002. This sample period spans six major recessions and six major expansions, several key historical and economic events that strongly affected U.S. interest rates, and covers the terms of four Federal Reserve chairmen, namely Burns, Miller, Volcker, and Greenspan. On the basis of a formal testing procedure, we show, not surprisingly, that the assumption of a constant covariance matrix is systematically rejected for U.S. interest rates over the past three decades. Interestingly, we find that common factors display a clear time-varying volatility over our sample period. Most notably, we observe that the switches in monetary policy that take place with the appointment of a new Federal Reserve chairman play an important role in characterizing the time variation in the loadings on the common factors that drive interest rates. Our empirical conclusions shed new light on the relation between the behavior of interest rates and shocks to the monetary policy (see, e.g., Mankiw and Miron 1986; Rudebusch 1995; Bernanke and Mihov 1998; Christiano, Eichenbaum, and Evans 1999; Bomfim 2003; Piazzesi 2005).

The remainder of the paper proceeds as follows. Section II describes the estimation, testing, and model selection procedures. Section III presents an empirical analysis based on the U.S. term structure of interest rates over the past three decades. Section IV offers a summary and concluding comments.

3. The policy inertia is defined by Piazzesi (2005) as positive autocorrelation in target rate changes. This positive autocorrelation is induced by the Federal Reserve's tendency to move its policy rate in a series of small steps. Monetary policy regimes have been shown to be associated with Federal Reserve chairmen (see Peek and Wilcox 1987).

II. Understanding Similarities among Covariance Matrices of Bond Yields

A. The General Framework

The approach taken in this paper is to break down the covariance matrix of bond yields at any point in time through principal component analysis. The term structure is defined as $X_t = (X_{1t}, X_{2t}, \ldots, X_{Mt})'$, where M is the number of maturities, and its associated covariance matrix is denoted Σ_t, $t = 1, \ldots, T$. We consider four cases of interest:

1. The matrix Σ_t may be assumed constant and equal to Σ for all t. In this case, both the eigenvectors and eigenvalues are required to be constant through time:

$$\Sigma = A\Lambda A', \quad (1)$$

where $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_M)$. The jth column of A gives the eigenvectors associated with the jth factor, and the diagonal elements of Λ give the eigenvalues, that is, the variances of the factors. This assumption is called the constant covariance matrix assumption.

2. The matrix Σ_t may be assumed to be proportional. This idea is formally expressed by the assumption that there exists a constant orthogonal matrix A of dimension $M \times M$ that jointly diagonalizes all covariance matrices Σ_t:

$$\Sigma_t = A\Lambda_t A', \quad (2)$$

where $\Lambda_t = \rho_t \times \Lambda_1$ and ρ_t are positive constants. This assumption is called the proportional covariance matrix assumption (see Flury 1988).

3. The matrix Σ_t may have constant eigenvectors but time-varying eigenvalues. This case differs from the proportional case in that the eigenvalues can freely vary through time:

$$\Sigma_t = A\Lambda_t A', \quad (3)$$

where $\Lambda_t = \mathrm{diag}(\lambda_{1t}, \ldots, \lambda_{Mt})$. This assumption is called the CPC assumption (see Flury 1984).

4. We may assume that all the eigenvectors and eigenvalues are time-varying. In this last case, covariance matrices at different points in time are assumed to be totally unrelated.

B. Maximum Likelihood Estimation

Suppose that the whole time period is divided into N consecutive subperiods of size $l_1, \ldots, l_N$ of multivariate observations X, with mean zero and covariance matrix Σ_n in the nth subperiod. Each random vector is now denoted $X_n = (X_{1n}, X_{2n}, \ldots, X_{Mn})'$, where M is the number of maturities. The first step in comparing two or more covariance matrices is creating a metric or statistic by which the comparison can be evaluated. A solution based on maximum likelihood methods has been known for some time for the covariance matrix equality assumption (see Anderson 1958). Basically, in this case, each separate covariance matrix is compared to the average of all the covariance matrices. The more different each covariance matrix is from the average, the less likely it is that the covariance matrices are equal to one another. Between the two extreme cases (equal and unrelated covariances), the proportional and the CPC assumptions offer new levels of similarities among subperiod covariance matrices.

If X_n is a sample from the M-variate normal distribution, $X_n \sim N(0, \Sigma_n)$, then the joint log likelihood function of $\Sigma_1, \ldots, \Sigma_N$ given the sample covariance matrices $S_1, \ldots, S_N$ is given by

$$\ln L(\Sigma_1, \ldots, \Sigma_N) = C - \frac{1}{2}\sum_{n=1}^{N} l_n[\ln \det \Sigma_n + \mathrm{tr}(\Sigma_n^{-1} S_n)], \tag{4}$$

where C is a constant term and tr denotes the trace operator (see Anderson 1958).

The maximum likelihood estimate of Σ under $H_0 : \Sigma_n^{\text{constant}} = \Sigma$ is given by the $M \times M$ pooled sample covariance matrix, $S = l^{-1}\sum_{n=1}^{N} l_n S_n$, where l is the total number of observations in the N subperiods. In this case, the number of parameters to be estimated is equal to $[M(M-1)/2] + M$.

Alternatively, the CPC assumption states that the sources of variation are constant through time, but their magnitude may differ across subperiods. In this case, the null assumption is $H_0 : \Sigma_n^{\text{CPC}} = A\Lambda_n A'$, where A is the $M \times M$ matrix of the eigenvectors and Λ_n is the diagonal matrix of eigenvalues $(\lambda_{1n}, \ldots, \lambda_{Mn})$ in the nth subperiod. In this case, the number of parameters is given by $[M(M-1)/2] + NM$.

Here, the challenge is to estimate the A and Λ_n matrices from the sample covariance matrices S_n, $n = 1, \ldots, N$. If we assume that the CPC framework is valid, Σ_n can be written as $A\Lambda_n A'$, and the joint log likelihood function given in equation (4) becomes

$$\ln L(\Sigma_1, \ldots, \Sigma_N) = C - \frac{1}{2}\sum_{n=1}^{N} l_n\{\ln \det (A\Lambda_n A') + \mathrm{tr}[(A\Lambda_n A')^{-1} S_n]\}. \tag{5}$$

Flury (1984) shows that the maximum likelihood estimate can be obtained by minimizing the following expression with respect to A:

$$\sum_{n=1}^{N} l_n[\ln \det \mathrm{diag}(A'S_n A) - \ln \det (A'S_n A)] \tag{6}$$

subject to the constraint $A'A = I$, where I is the unit matrix. Minimizing this function can be viewed as trying to find a matrix A, which diagonalizes jointly the matrices S_n, $n = 1, \ldots, N$, as much as it can. A numerical algorithm can be found in Flury (1988, app. C).

The estimation of A and Λ_n under the proportional covariance matrix assumption $H_0 : \Sigma_n^{\text{prop}} = \rho_n A\Lambda_1 A'$ differs from the CPC case by imposing

$\lambda_{mn} = \rho_n \lambda_{m1}$ (see Flury 1988, 103). This additional constraint limits the number of parameters to $[M(M-1)/2] + M + N - 1$.

Finally, when the covariance matrices in the N subperiods are assumed to be totally unrelated, the number of parameters to be estimated increases to $NM(M+1)/2$.

C. *Log Likelihood Ratios and Model Selection*

The assumptions presented above can be ordered in a hierarchical fashion, which allows a detailed analysis of the involved covariance matrices of different subperiods. The highest level of similarity would be to assume equality between covariance matrices of different subperiods (constant covariance assumption). In this case, one may obtain the parameters by running a single principal component applied to the pooled sample covariance matrix of all N subperiods. The assumptions subsequently relaxing the restrictions are the proportional covariance and CPC assumptions. The relations between different subperiod covariances disappear subsequently, until at the last level the covariance matrices do not share any common eigenstructure.

The usual log likelihood ratio statistics (T) can be computed to test any assumption against the unrelated covariance assumption. For instance, for the constant covariance assumption, the test is

$$T_{\text{constant vs. unrelated}} = -2 \ln \frac{L(S, \ \ldots, \ S)}{L(S_1, \ \ldots, \ S_N)} = l \ln \det S - \sum_{n=1}^{N} l_n \ln \det S_n, \qquad (7)$$

where $L(S_1, \ \ldots, S_N)$ (respectively, $L(S, \ \ldots, S)$) is the unrestricted (respectively, restricted to constant matrix) maximum of the likelihood function. The statistic is asymptotically χ^2 with $(N-1)\{[M(M-1)/2] + M\}$ degrees of freedom.[4] Similarly for the other assumptions, we get

$$T_{\text{prop vs. unrelated}} = \sum_{n=1}^{N} l_n \ln \det \hat{\Sigma}_n^{\text{prop}} - \sum_{n=1}^{N} l_n \ln \det S_n \qquad (8)$$

and

$$T_{\text{CPC vs. unrelated}} = \sum_{n=1}^{N} l_n \ln \det \hat{\Sigma}_n^{\text{CPC}} - \sum_{n=1}^{N} l_n \ln \det S_n, \qquad (9)$$

which are asymptotically χ^2 with $(N-1)[(M^2 + M - 2)/2]$ and $(N-1)[M(M-1)/2]$ degrees of freedom, respectively.[5]

As all these different models are nested, one can decompose the total T-

4. Degrees of freedom of the corresponding T-test are obtained by subtracting the number of parameters to be estimated in the two models under comparison.

5. We denote by $\hat{\Sigma}_n$ the maximum likelihood estimate of Σ_n.

statistic into partial T-statistics and test any assumption against a less restrictive one. By the summation property, a test of assumption A against any hierarchically lower assumption B is given by subtracting $T_{B\,\text{vs. unrelated}}$ from $T_{A\,\text{vs. unrelated}}$. The obtained statistic is asymptotically χ^2, and the number of degrees of freedom is again given by the difference in the number of parameters between the two models. As a result, the T-statistic contrasting the constant covariance and the proportional covariance assumptions is asymptotically χ^2 with $N - 1$ degrees of freedom; the T-statistic contrasting the proportional covariance and the CPC assumptions is asymptotically χ^2 with $(M - 1)(N - 1)$ degrees of freedom.

While likelihood ratio tests can naturally be constructed to discriminate among the candidate assumptions, one can also use the Akaike information criterion (AIC) to select the most appropriate model. The AIC balances the goodness of fit of a particular model, that is, the log likelihood value, against the number of parameters to be estimated. As models with more parameters tend to fit better out of necessity, the best model in this scheme is chosen using a penalized log likelihood controlling for the number of parameters. The AIC is defined as

$$\text{AIC} = -2(\text{maximum of log likelihood}) + 2(\text{number of parameters}),$$

and the model with the lowest value for the AIC is the best-fitting one.

III. Empirical Analysis

A. Data and First Results

We now apply the methodology presented in the previous section to the U.S. term structure of interest rates from January 1970 through December 2002. We use the Fama-Bliss (1987) monthly data on Treasury zero-coupon bond yields with the following maturities: 3, 6, 12, 24, 60, and 120 months.[6] Maturities shorter than three months are not included in the sample because of possible liquidity problems (see Duffee 1996). Since a hump in the volatility curve is encountered at the two-year maturity, this intermediate maturity plays an important role in any study of the volatility of interest rates. We also include the 10-year bond yield partly because it is an important benchmark in the bond market and partly because whether or not a model can fit a 10-year maturity is a much more discriminating criterion than shorter maturities.[7]

According to the National Bureau of Economic Research (NBER), this sample period contains six major recessions and six major expansions.[8] Several key historical and economic events occurred during our period of analysis

6. We thank Robert Bliss for providing us with the bond yield data.

7. We thank the referee for this remark.

8. The NBER peaks are 1969 (IV), 1973 (IV), 1980 (I), 1981 (III), 1990 (III), and 2001 (I); the NBER troughs are 1970 (IV), 1975 (I), 1980 (III), 1982 (IV), 1991 (I), and 2001 (IV). Quarterly dates are in parentheses.

TABLE 1 Descriptive Statistics (January 1970–December 2002)

	Mean	Standard Deviation	Skewness	Excess Kurtosis	$\rho(1)$
			A. Bond Yields		
3 months	6.49	2.78	1.05	1.43	.971
6 months	6.71	2.81	.97	1.19	.973
12 months	6.92	2.73	.85	.91	.971
24 months	7.20	2.59	.85	.78	.974
60 months	7.62	2.35	.92	.54	.979
120 months	7.85	2.21	.97	.55	.983
			B. Bond Yield Changes		
3 months	−.017	.61	−1.73	13.46	.128
6 months	−.017	.59	−1.41	12.42	.162
12 months	−.017	.58	−1.10	13.32	.152
24 months	−.016	.51	−.45	8.61	.176
60 months	−.013	.41	−.11	2.67	.125
120 months	−.009	.36	−.17	1.45	.098

NOTE.—Descriptive statistics are computed from the 396 monthly observations from 1970:01-2002:12 for bond yields and bond yield changes, for maturities ranging from three months to 10 years. $\rho(1)$ is first-order autocorrelation.

(e.g. the oil price shocks, the monetary experiment, the 1987 crash, and the Gulf War), among which some strongly affected U.S. interest rates.[9] Moreover, in this sample, there have been four different Federal Reserve chairmen (see Thornton [1996] for more details): Arthur F. Burns (February 1970–January 1978), G. William Miller (March 1978–August 1979), Paul A. Volcker (August 1979–August 1987), and finally Alan Greenspan (August 1987–present). We see in table 1 that, over the past three decades, the bond yields increase, on average, with maturity: the term structure is upward sloping. The volatility of bond yields is globally decreasing with a hump at six months. While bond yields are highly autocorrelated (around 0.980), bond yield *changes* appear to be far less persistent (around 0.150). Since time dependence affects our testing procedure, we use bond yield changes in the following empirical analysis.

We report in figure 1 the term structure of volatility of bond yield changes for different sample periods. As pointed out by Dai and Singleton (2003) and Piazzesi (2003), the volatility curve may look really different in successive time periods. We associate different subperiods with the terms of Federal Reserve chairmen and also distinguish between periods of low volatility, such as the Greenspan era, and periods of very high volatility, such as the 1979–82 monetary experiment. We observe that the shape of the volatility curve varies substantially across the terms of Federal Reserve chairmen. More precisely, while for the Greenspan era the volatility curve appears to be hump-shaped, this is not a phenomenon observed over other eras of Federal Reserve chairmen

9. The monetary experiment corresponds to the period from October 1979 to October 1982 during which the Federal Reserve focused primarily on reducing the rate of growth of monetary aggregates, rather than targeting interest rates, in an effort to reduce inflation.

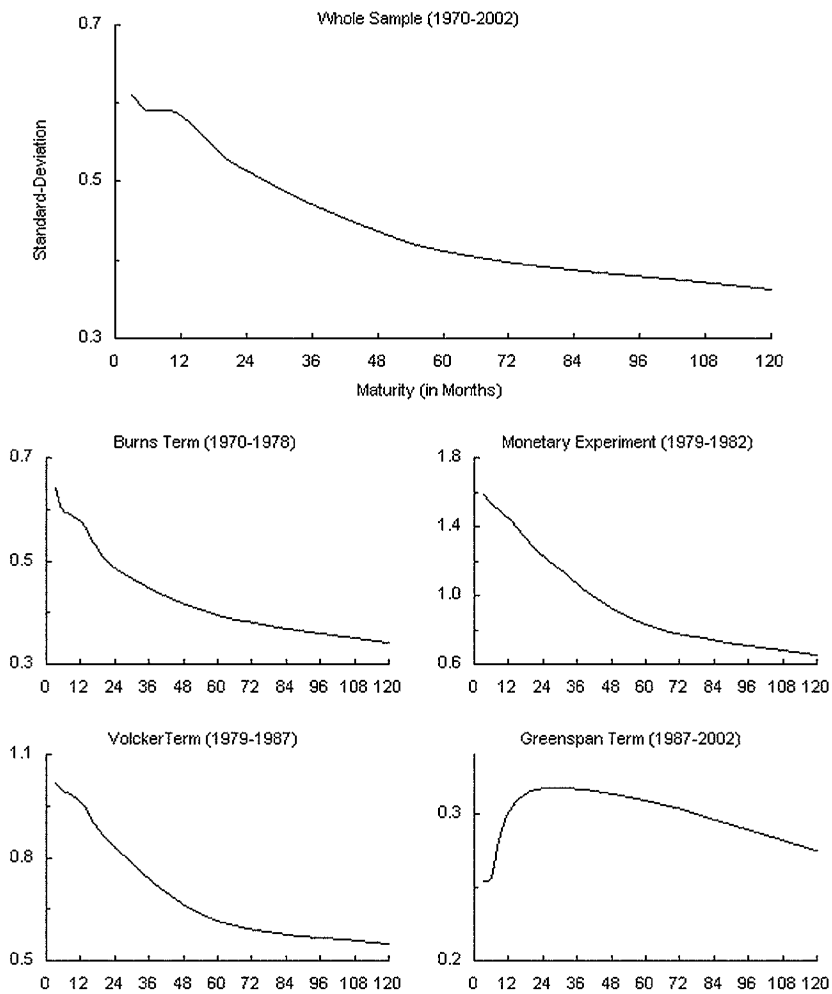

FIG. 1.—Volatility curves in different time periods. This figure presents the value of the standard deviations of bond yield changes, for maturities ranging from three months to 10 years, in different time periods, respectively, the whole sample period (1970:01–2002:12), the monetary experiment (1979:10–1982:10), and the terms of three Federal Reserve chairmen: Burns (1970:02–1978:01), Volcker (1979:08–1987:07), and Greenspan (1987:08–2002:12).

or during the monetary experiment. In order to assess the influence of the latter subperiod, we partition our total sample into three subperiods in order to embrace the monetary experiment and run a separate principal component analysis in each subperiod. The first period is from January 1970 through September 1979, the second one from October 1979 through October 1982, and the third one from November 1982 through December 2002. We run a

TABLE 2 Eigenvectors and Eigenvalues

	A. Full Sample								
	Prior to the Monetary Experiment			Monetary Experiment			After the Monetary Experiment		
	PC1	PC2	PC3	PC1	PC2	PC3	PC1	PC2	PC3
a_{3M}	.507	−.614	−.394	.507	−.637	−.270	.303	−.610	−.449
a_{6M}	.503	−.235	.158	.505	−.220	−.055	.373	−.415	−.135
a_{12M}	.490	.157	.394	.479	.190	.531	.449	−.165	.413
a_{24M}	.380	.420	.264	.398	.385	.282	.464	.085	.455
a_{60M}	.273	.447	−.151	.253	.467	−.276	.443	.375	.087
a_{120M}	.173	.410	−.757	.189	.380	−.698	.395	.530	−.629
Eigenvalues	1.00	.12	.04	8.78	.42	.08	.53	.08	.01
% variance	83.9	9.9	3.1	93.7	4.4	.8	83.2	12.6	2.1
	B. Federal Reserve Chairman Terms								
	Burns Term			Volcker Term			Greenspan Term		
	PC1	PC2	PC3	PC1	PC2	PC3	PC1	PC2	PC3
a_{3M}	.512	−.605	−.397	.486	−.581	−.404	.299	−.605	−.491
a_{6M}	.507	−.233	.158	.495	−.247	.056	.356	−.417	.121
a_{12M}	.486	.153	.384	.478	.087	.491	.453	.193	.451
a_{24M}	.376	.427	.273	.405	.273	.406	.490	.109	.408
a_{60M}	.272	.444	−.132	.277	.482	−.119	.456	.390	.018
a_{120M}	.172	.421	−.761	.222	.536	−.643	.363	.509	−.612
Eigenvalues	1.09	.13	.04	3.93	.29	.05	.40	.07	.01
% variance	83.6	10.0	3.3	91.0	6.6	1.2	81.9	13.7	2.3

NOTE.—This table displays the eigenvectors (a_m, $m = 3$, 6, 12, 24, 60, and 120 months) and the eigenvalues of the first three factors estimated from subperiod covariance matrices of bond yield changes, with the percentage of the total variance of the original data (% variance) captured by each factor. In panel A, the three subperiods covered are the period prior to the monetary experiment (1970:01–1979:09), the monetary experiment (1979:10–1982:10), and after the monetary experiment (1982:11–2002:12). In panel B the subperiods coincide with the Federal Reserve chairmanships of Burns (1970:02–1978:01), Volcker (1979:08–1987:07), and Greenspan (1987:08–2002:12).

similar principal component analysis by associating different subperiods with the terms of Federal Reserve chairmen. In table 2 what stands out is the really high variability of the variance of the common factors across subperiods. As far as factor loadings are concerned, it seems difficult to reach any definite conclusion by simply eyeballing the estimated eigenvectors in table 2. Consequently, an appropriate statistical test, such as the one presented below, appears necessary to conclude positively.

B. Empirical Results

In order to disentangle the sources of time variation in the covariance matrix of interest rates, we compare the following alternative assumptions: constant covariance, proportional covariance, CPC, and unrelated covariance assumptions. We analyze for each assumption the log likelihood ratios and the AIC.[10]

10. To ease the presentation, we report in table 3 the modified AIC proposed by Flury (1988, 153). If we assume that we have I models to compare, that model i has p_i parameters, and that $p_1 \leq p_i \leq p_I$, the modified AIC for model i is given by

Our tests are first run with three subperiods suggested by the exploratory results found in table 2, that is, prior to the monetary experiment, January 1970–September 1979; during the monetary experiment, October 1979–October 1982; and after the monetary experiment, November 1982–December 2002 (panel A of table 3). In order to control for the most volatile episode of the history of U.S. interest rates, we conduct a similar analysis by excluding the monetary experiment and considering two subperiods only (panel B). According to these first two panels, both the log likelihood ratios and the AIC reject the constant covariance assumption and then attest that eigenvalues are time-varying. It is important to point out that these results arise whether or not an exceptionally volatile episode is present in the sample. Concerning the factor loadings, time variation is encountered when the whole time period is considered, but also when the monetary experiment is dropped.

Since U.S. monetary policy regimes are usually associated with Federal Reserve chairmen (see Peek and Wilcox 1987; Piazzesi 2005), we also consider alternative partitions of the whole sample by associating different subperiods with the terms of Federal Reserve chairmen. We study the evolution through time of the factor structure within two successive chairman terms (panel C, Burns and Volcker; panel D, Volcker and Greenspan) and also within each chairmanship (panels E–G).[11] Specifically, we divide each chairman's term into two equal subperiods. For instance, the first subperiod of the Greenspan term covers August 1987–October 1994, and the second one covers November 1994–December 2002.

Regardless of the terms considered, our formal statistical procedure clearly rejects the constant covariance matrix assumption. Indeed, log likelihood ratios systematically reject the equal covariance assumption against the unrelated covariance matrix assumption (p-value $< .05$), and the AIC systematically chooses some covariance decompositions allowing eigenvalues to vary through time. We note that, while it is not surprising that these results arise when an exceptionally volatile episode is present in the sample, they remain valid when one considers more stable or homogeneous periods, such as the Greenspan term.

Moreover, while, over successive chairman terms, factor loadings are time-varying, they appear to be constant within a given chairmanship. More precisely, over the Burns-Volcker terms and Volcker-Greenspan terms, log likelihood ratio tests systematically reject the proportional covariance and the CPC assumptions against the unrelated covariance assumption (see panels C

$$\mathrm{AIC}(i) = -2(\ln L_i - \ln L_l) + 2(p_i - p_1),$$

where L_i is the maximum of the likelihood function of model i. Selecting the model with the lowest AIC is equivalent to selecting the model with the lowest AIC(i). This modified AIC has the advantage of being directly related to the likelihood ratio statistic and the number of parameters.

11. Because of its short length, the Miller term (1978:03–1979:07) is not analyzed as a separate subperiod.

TABLE 3 Likelihood Ratios and Model Selection

H_0	$H_{alternative}$	T	Degrees of Freedom	p-Value	AIC
			A. Full Sample		
Equal	Unrelated	832.19	42	.000	832.19
Proportional	Unrelated	184.18	40	.000	188.18
CPC	Unrelated	148.27	30	.000	172.27
Equal	Proportional	648.01	2	.000	. . .
Proportional	CPC	35.91	10	.000	. . .
Unrelated	. . .	. . .	. . .	. . .	84.00*
			B. Without Monetary Experiment		
Equal	Unrelated	309.00	21	.000	309.00
Proportional	Unrelated	116.42	20	.000	118.42
CPC	Unrelated	97.32	15	.000	109.32
Equal	Proportional	192.58	1	.000	. . .
Proportional	CPC	19.11	5	.002	. . .
Unrelated	. . .	. . .	. . .	. . .	42.00*
			C. Burns and Volcker Terms		
Equal	Unrelated	94.75	21	.000	94.75
Proportional	Unrelated	58.59	20	.000	60.59
CPC	Unrelated	30.44	15	.010	42.44
Equal	Proportional	36.15	1	.000	. . .
Proportional	CPC	28.16	5	.000	. . .
Unrelated	. . .	. . .	. . .	. . .	42.00*
			D. Volcker and Greenspan Terms		
Equal	Unrelated	658.44	21	.000	658.44
Proportional	Unrelated	107.80	20	.000	109.80
CPC	Unrelated	88.74	15	.000	100.74
Equal	Proportional	550.64	1	.000	. . .
Proportional	CPC	19.06	5	.002	. . .
Unrelated	. . .	. . .	. . .	. . .	42.00*
			E. Burns Term		
Equal	Unrelated	45.78	21	.001	45.78
Proportional	Unrelated	44.90	20	.001	46.90
CPC	Unrelated	27.82	15	.023	39.82*
Equal	Proportional	.87	1	.350	. . .
Proportional	CPC	17.09	5	.004	. . .
Unrelated	. . .	. . .	. . .	. . .	42.00
			F. Volcker Term		
Equal	Unrelated	189.84	21	.000	189.84
Proportional	Unrelated	57.91	20	.000	59.91
CPC	Unrelated	40.90	15	.000	52.90
Equal	Proportional	131.92	1	.000	. . .
Proportional	CPC	17.01	5	.005	. . .
Unrelated	. . .	. . .	. . .	. . .	42.00*
			G. Greenspan Term		
Equal	Unrelated	36.19	21	.021	36.19
Proportional	Unrelated	36.02	20	.015	38.02
CPC	Unrelated	22.50	15	.095	34.50*
Equal	Proportional	.17	1	.684	. . .

TABLE 3 (*Continued*)

H_0	$H_{alternative}$	T	Degrees of Freedom	p-Value	AIC
Proportional	CPC	13.52	5	.019	. . .
Unrelated	. . .	. . .	. . .	. . .	42.00

NOTE.—In each panel, assumptions on covariance matrices are tested, starting from equal covariance matrices, then proportional covariance matrices, common principal component (CPC), and ending with unrelated covariance matrices. T denotes the log likelihood ratio statistics testing the assumption H_0 against $H_{alternative}$. In panel A, the three subperiods covered are the period prior to the monetary experiment (1970:01–1979:09), the monetary experiment (1979:10–1982:10), and after the monetary experiment (1982:11–2002:12). In panel B, the two subperiods covered are the period prior to the monetary experiment (1970:01–1979:09) and the period after the monetary experiment (1982:11–2002:12). In panels C–D, the two subperiods coincide with the terms of two Federal Reserve chairmen: Burns (1970:02–1978:01), Volcker (1979:08–1987:07), and Greenspan (1987:08–2002:12). In panels E–G, two equal subperiods cover the term of a single Federal Reserve chairman.

* The chosen assumption (H_0) according to the AIC.

and D), and the AIC reaches its minimum for the unrelated covariance assumption. Over the Greenspan chairmanship, a different picture emerges since the CPC assumption cannot be rejected against the unrelated covariance matrix assumption (p-value of .095). At the same time, the even more parsimonious proportional assumption cannot be rejected against the CPC one at the 1% level but is rejected at the 5% level (p-value of .023). In this sample period, the AIC chooses the CPC decomposition of the covariance matrix. Over the Burns term, although the evidence is less clear-cut, the winning assumption is also the CPC one. The only exception turns out to be the Volcker term (panel F), and this should not come as a surprise. Shortly after being appointed by President Carter in August 1979, Volcker initiated an anti-inflation strategy based on the monetarist theory, which called on the Federal Reserve to target the supply of money rather than interest rates, and ended it in 1982. The monetary policy conducted by Volcker during the second part of his term was radically different. The results obtained for the Volcker term reinforce our conclusion stating that as long as the monetary policy is not significantly changed, the factor loadings remain constant.

IV. Conclusion

In this paper, we show that understanding the sources of time variation in the covariance matrix of interest rates within the context of a principal component analysis greatly increases the set of questions that can be addressed in the study of interest rates. On the basis of a formal testing procedure, we show, not surprisingly, that the assumption of a constant covariance matrix is systematically rejected for U.S. interest rates over the past three decades. Interestingly, we find that common factors display a clear time-varying volatility over our sample period. Most notably, we observe that the switches in monetary policy that take place with the appointment of a new Federal Reserve chairman play an important role in characterizing the time variation in the loadings on the common factors that drive interest rates.

Our results have important implications for the study of the links between interest rates and the macroeconomy (see Ang and Piazzesi 2003; Hördahl, Tristani, and Vestin 2004). Indeed, time variation in the factor structures of interest rates and macroeconomic variables critically affects the estimation of these links. To avoid misleading results, researchers should conduct such studies over stable time periods, as in Rudebusch and Wu (2003), or develop methodologies to deal with unstable factor structures. The methodology proposed in this paper can also be used to assess the stability of the factor structure of macroeconomic variables.

While this paper establishes empirically that different monetary policy regimes lead to different behavior in the second-moment properties of the term structure, more research needs to be done to understand the main transmission channels of the monetary policy. Furthermore, while we focus on the government bond yield factor structure, corporate bond spreads (see Collin-Dufresne, Goldstein, and Martin 2001), money market returns (see Knez, Litterman, and Scheinkman 1994), or covariance matrices implied from swaption prices (see Longstaff, Santa-Clara, and Schwartz 2001) may also be investigated using the methodology proposed in the present paper. This investigation is left for further research.

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